Pricing Implications of Conditional Market Beta

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Motivation

- CAPM is rejected unconditionally
  - Different factors in asset pricing model
  - Conditional CAPM
- Conditional CAPM (CCAPM)
  - Moving average of beta (Fama-MacBeth, 1973)
  - Beta as function of macroeconomic variables
  - But CCAPM is unlikely to explain stock returns (Lewellen and Nagel, 2006)
- Engle (2014) constructs dynamic conditional beta through multivariate GARCH model
  - Make conditional beta observable
  - Conditional betas vary across the time
  - Conditional CAPM fit the data better than unconditional CAPM

Research Questions

- Does dynamic conditional beta affect asset pricing?
  - Compare with other factors in popular asset pricing models
- Do shocks to beta serve as a state variable in the ICAPM setting?
  - The unexpected shocks to the conditional beta may affect investment opportunity set
- If shocks to conditional beta affect pricing, can we use them as a pricing factor to price other assets?

Preview

- We use a multivariate GARCH approach to construct the conditional market beta for different assets (Engle, 2014)
  $\rightarrow$ For each test asset we have a time-series of conditional beta
- We use Principle Component Analysis (PCA) to extract the covariation of conditional betas
  - PCA on the unexpected shocks to the conditional betas
  - Retain the PCs that capture covariation (1 PC for Momentum; 2 PCs for Size and Book-to-market)
- Construct factor mimicking portfolios on each PC
  - Form a tradable portfolio that is a bet on each PC factor
  - Portfolios that capture the unexpected shocks to the conditional betas

Preview

- There is some predictability of PC factors in market returns and market volatilities
- The correlation between PC factors and Fama-French is large
- PC factors earns large abnormal returns according to popular factor pricing models $\rightarrow$ pricing factors miss important risks
- Alphas in CAPM is larger than alphas FF4$\rightarrow$ some overlap between Fama-French factors and PCs
- As a pricing factor PC factors can explain some of the data compared to other popular asset pricing models

Settings

- Conditional CAPM:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,t}(r_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

- Conditional market beta in CAPM setting:

$$\beta_{i,t}^{DCC} = \frac{\sigma_{im,t}}{\sigma_{m,t}^2} = \rho_{im,t}\frac{\sigma_{i,t}}{\sigma_{m,t}}$$

where $\sigma_{im,t}$ is the conditional covariance between asset return and market return, $\sigma_{m,t}^2$ is the conditional variance of market return, and $\rho_{im,t}$ is the conditional correlation between asset return and market return

Settings

- GARCH(1,1) model for conditional variance:

$$\sigma_{i,t}^2 = \overline{\sigma_i^2} + \theta_{i,1}(\sigma_{i.t-1}^2 - \overline{\sigma_i^2}) + \theta_{i,2}(r_{i.t-1}^2 - \overline{\sigma_i^2}),\ \ \theta_{i,1} + \theta_{i,2} < 1$$

- GARCH(1,1) model for conditional correlation:

$$q_{ij,t} = \overline{\rho_{ij}} + \phi_{i,1}(q_{ij,t-1} - \overline{\rho_{ij}}) + \phi_{i,2}(\epsilon_{1,t-1}\epsilon_{2,t-1} - \overline{\rho_{ij}}),\ \phi_{i,1} + \phi_{i,2} < 1$$

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$$

where

$$r_{i,t} = \sigma_{i,t}\epsilon_{i,t}, \qquad r_{j,t} = \sigma_{j,t}\epsilon_{j,t},$$

$$\epsilon_{i,t} \sim N(0,1), \qquad \epsilon_{j,t} \sim N(0,1)$$

Settings

- Conditional convariance matrix of returns:

$$H_t = D_t R_t D_t$$

where R is the time-varying conditional correlation matrix and

$$D_t = diag\{\sigma_{i,t}\}$$

- Maximum Likelihood Estimation (MLE):

$$r_t \sim N(0, H_t)$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(nlog(2\pi) + log|H_t| + r_t' H_t^{-1} r_t)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}(nlog(2\pi) + 2log|D_t| + r_t' D_t^{-2} r_t - \epsilon_t' \epsilon_t + log|R_t| + \epsilon_t' R_t^{-1} \epsilon_t)$$

Settings

- Write the loglikelihood as the sum of a volatility part and a correlation part:

$$L(\theta, \phi) = L_v(\theta) + L_C(\theta, \phi)$$

$$L_v(\theta) = -\frac{1}{2} \sum_{t=1}^{T} (nlog(2\pi) + 2log|D_t| + r_t' D_t^{-1} D_t^{-1} r_t)$$

$$L_C(\theta, \phi) = -\frac{1}{2} \sum_{t=1}^{T} (log|R_t| + \epsilon_t' R_t^{-1} \epsilon_t - \epsilon_t' \epsilon_t)$$

- The two-step approach to maximizing the likelihood:

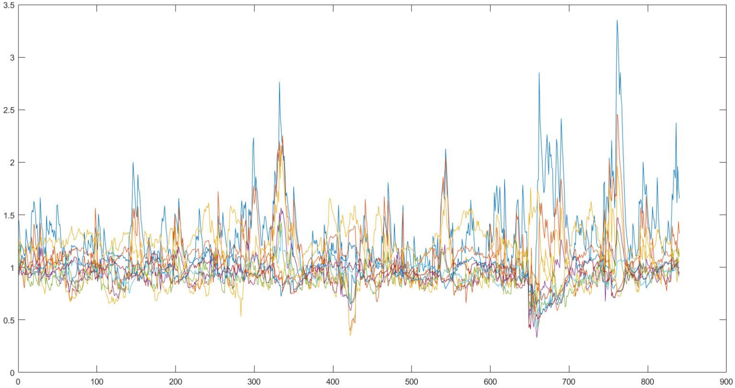
$$\theta^* = argmax\{L_v(\theta)\}$$

$$max_\phi\{L_C(\theta^*, \phi)\}$$

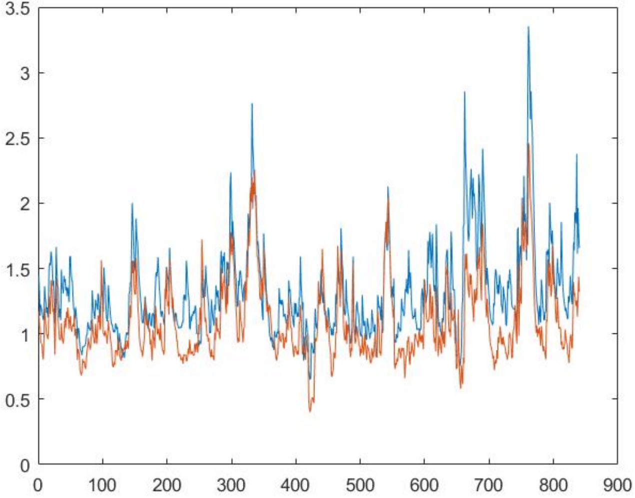
Data

- Monthly returns of 25 portfolios formed on size and book-to-market from Jun 1926 to Aug 2016
- Monthly returns of 10 portfolios formed on momentum from Jun 1926 to Aug 2016
- Monthly data of Fama-French four factors from Jun 1926 to Aug 2016
- Other data including portfolios formed on industry, operating profitability, investment, short-term reversal, long-term reversal, size, book-to-market, and Novy-Marx's 320 deciles portfolios

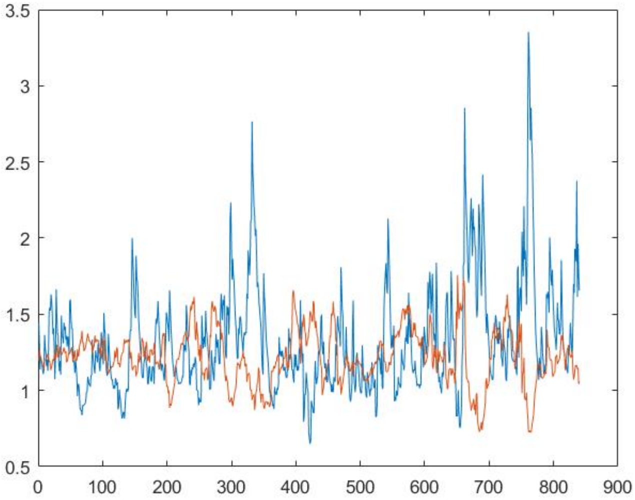
Conditional Beta of 10 Momentum Portfolios



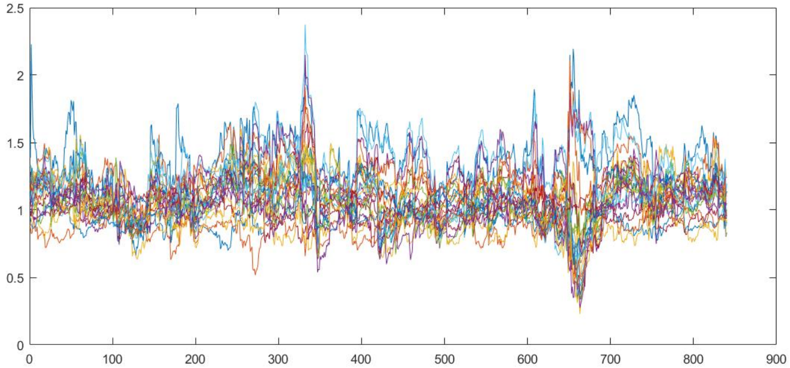
Conditional Beta of 1st and 2nd Momentum Portfolios



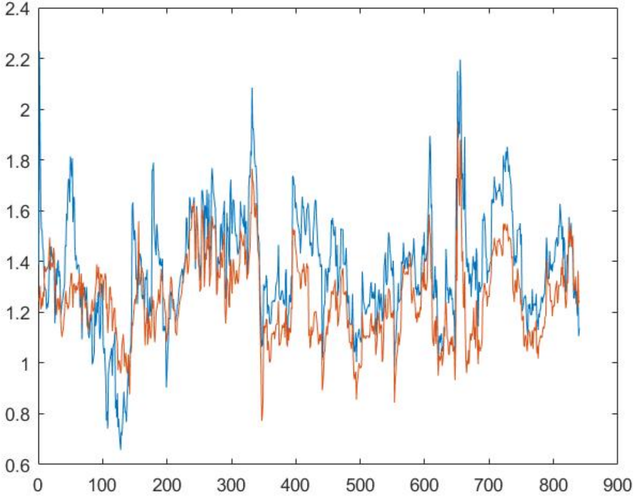
Conditional Beta of 1st and 10th Momentum Portfolios



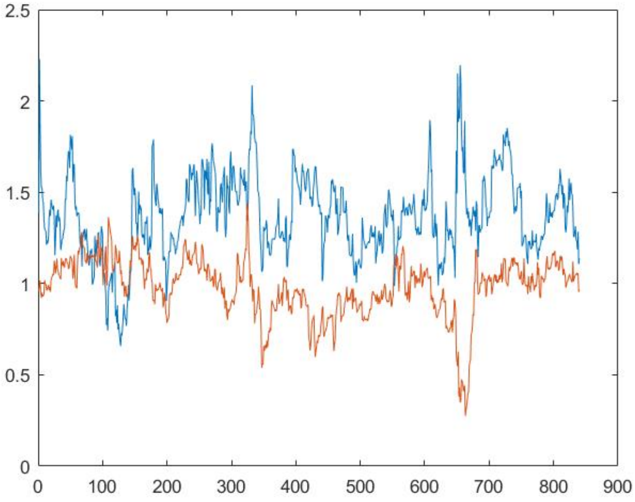
Conditional Beta of 25 Size and Book-to-market Portfolios



Conditional Beta of 1st and 2nd Size and Book-to-market Portfolios



Conditional Beta of 1st and 25th Size and Book-to-market Portfolios



Principal Component Analysis

- Use PCA to extract the covariation of the conditional market betas
- We are interested in the unexpected shocks of betas
- Focus on the change of conditional market betas

Principal Component Analysis

- Use AR(1) Model to get unexpected shocks on beta $d\beta^*$:

$$\beta_{t+1} - \beta_t = \lambda(\beta_t - \beta_{t-1}) + \epsilon_{t+1}$$

$$d\beta^*_{t+1} = \epsilon_{t+1}$$

- PCA on $d\beta^*$ to transform to a new set of variables, the principal components (PCs)
  $\rightarrow$ Uncorrelated and ordered so that the first few retain most of the variation retained in all of the original variables

Principal Component Analysis

- Construct N principal components (PCs) of N conditional unexpected shocks of beta:

$$PC = d\beta^* W$$

  where W,Λ: $N \times N$ matrices of eigenvectors, eigenvalues of $N \times N$ covariance matrix Σ, with $\Sigma W = W\Lambda$, and Σ is the covariance matrix of $d\beta^*$

- Variation explained by the i-th PC:

$$Variation_i = \frac{\lambda_i}{\sum_{k=1}^{N} \lambda_k}$$

- Use Parallel Analysis (PA Test) to determine the components capturing common variation retained from PCA
  - Based on Monte-Carlo simulation and retained if bigger than 95th of the distribution derived from random data

Factor Mimicking Portfolios

- Form a tradable portfolio that is a bet on each PC factor:

$$PC_{i,t} = \alpha_i + \sum_{j=1}^{N} \eta_{i,j} R_{j,t} + \epsilon_{i,t}$$

  where η is portfolio weights on N excess returns of test assets, and R is the returns of the assets

- Use the return of factor mimicking portfolios as proxies of PC factors:

$$RPC_{i,t} = \sum_{j=1}^{N} \eta_{i,j} R_{j,t}$$

- The factor mimicking portfolios are constructed by a buy-and-hold strategy $\rightarrow$ the weights do not change throughout the time

10 Momentum Porfolios PCs, 1946-2016

Momentum	PC1	PC2	PC3	PC4
Variation explained(%)	63.12	16.41	5.56	4.16
Common Variation	Yes	No	No	No
Corr(RPC,Mkt)	-0.01	0.32	0.18	0.52
Corr(RPC,SMB)	0.03	0.34	-0.08	0.43
Corr(RPC,HML)	-0.18	-0.13	0.14	-0.29
Corr(RPC,MOM)	0.91	-0.34	-0.28	-0.15

25 Size and Book-to-market Portfolios PCs, 1946-2016

Size and Book-to-market	PC1	PC2	PC3	PC4
Variation explained(%)	50.98	16.17	5.07	3.69
Common Variation	Yes	Yes	No	No
Corr(RPC,Mkt)	-0.07	0.03	-0.02	-0.08
Corr(RPC,SMB)	0.56	-0.45	0.37	-0.52
Corr(RPC,HML)	-0.16	0.52	-0.25	-0.05
Corr(RPC,MOM)	0.03	-0.18	0.01	0.00

Abnormal Returns of Factor Mimicking Portfolios

- Unconditional CAPM:

$$RPC_{i,t} = \alpha_i + \beta_i Mkt_t + \epsilon_{i,t}$$

- Unconditional FF4:

$$RPC_{i,t} = \alpha_i + \beta_{i,1} Mkt_t + \beta_{i,2} SMB_t + \beta_{i,3} HML_t + \beta_{i,4} MOM_t + \epsilon_{i,t}$$

Unconditional CAPM Abnormal Returns

Momentum	PC1	PC2	PC3	PC4
alpha	12.81***	-3.73***	-2.77***	-0.03
(t-test)	6.51	-5.27	-4.42	-0.13
beta	-0.02	0.14***	0.06***	0.09***
(t-test)	-0.20	7.19	3.73	12.27
R^2	0.00	0.11	0.03	0.27

FF4 Abnormal Returns

Momentum	PC1	PC2	PC3	PC4
alpha	0.99	-1.99***	-2.19***	0.40*
(t-test)	1.14	-2.64	-3.03	1.65
Mkt	0.11***	0.09***	0.07***	0.07***
(t-test)	4.65	5.13	4.44	10.04
SMB	-0.03	0.17***	-0.06**	0.08***
(t-test)	-0.58	6.10	-2.52	4.94
HML	-0.03	-0.06	0.07***	-0.05***
(t-test)	-0.59	-1.81	2.61	-3.65
MOM	1.22***	-0.15***	-0.09***	-0.02***
(t-test)	39.08	-6.75	-4.20	-3.01
R^2	0.83	0.27	0.13	0.39

Unconditional CAPM Abnormal Returns

Size and Book-to-market	PC1	PC2	PC3	PC4
alpha	-1.85	4.17***	-1.72***	-0.58
(t-test)	-0.7	2.73	-3.05	-1.07
beta	-0.11	0.02	-0.01	-0.02
(t-test)	-1.53	0.65	-0.63	-1.48
R^2	0.01	0.00	0.00	0.01

FF4 Abnormal Returns

Size and book-to market	PC1	PC2	PC3	PC4
alpha	-1.33	1.89	-1.01*	0.03
(t-test)	-0.59	1.54	-1.74	0.07
Mkt	-0.40***	0.20***	-0.06***	0.01
(t-test)	-5.99	5.75	-4.97	0.75
SMB	1.43***	-0.54***	0.19***	-0.25***
(t-test)	8.84	-4.07	6.25	-7.22
HML	-0.24	0.64***	-0.12***	-0.07**
(t-test)	-1.55	7.53	-4.07	-2.10
MOM	0.02	-0.09	-0.01	-0.01
(t-test)	0.20	-1.38	-0.54	-0.57
R^2	0.38	0.46	0.19	0.3

Predictability of PC Factors

- Run predictive regressions on each factor
- Market Returns:

$$R_{Mkt,t+T} = \alpha_{Mkt} + \sum_{j=1}^{n} \beta_{Mkt,j} F_{j,t} + \epsilon_{Mkt,t}$$

- Market Volatilities:

$$Vol_{Mkt,t+T} = \alpha_{Mkt} + \sum_{j=1}^{n} \beta_{Mkt,j} F_{j,t} + \epsilon_{Mkt,t}$$

Predictability of Market Returns

	T=1	T=6	T=12	T=24	T=36
Mkt	0.07111*	0.01848	0.0108	-0.0048	-0.0035
(R^2)	0.3800	0.0600	0.0000	-0.0800	-0.0800
SMB	0.0547	-0.04561*	-0.0239	-0.0179	-0.01786*
(R^2)	0.0100	0.3600	0.1400	0.1800	0.3800
HML	-0.08613*	-0.0452	-0.0246	-0.0136	-0.0024
(R^2)	0.1800	0.3100	0.1300	0.0400	-0.1200
MOM	-0.0111	-0.03238**	-0.02651**	-0.01879**	-0.0106
(R^2)	-0.1100	0.3300	0.4700	0.5100	0.2100

Predictability of Market Returns

	T=1	T=6	T=12	T=24	T=36
Mkt	0.07111*	0.01848	0.0108	-0.0048	-0.0035
(R^2)	0.3800	0.0600	0.0000	-0.0800	-0.0800
MPC1	0.0048	-0.0011	0.0014	0.0001	-0.0010
(R^2)	-0.0700	-0.1100	-0.0700	-0.1200	-0.0400
MRPC1	-0.0185	-0.02563*	-0.02058**	-0.01585**	-0.00979*
(R^2)	-0.0700	0.3800	0.5100	0.6700	0.3800
SPC1	0.0045	0.00562**	0.0014	0.0018	0.0000
(R^2)	-0.0800	0.2300	-0.0800	0.0200	-0.1300
SRPC1	-0.0025	-0.0101	-0.0052	-0.0046	-0.00988*
(R^2)	-0.1200	0.0100	-0.0500	-0.0100	0.7400

Predictability of Market Volatilities

	T=0	T=0.5	T=1	T=2	T=3
Mkt	0.0003	-0.0019	-0.0024**	-0.0021***	-0.0014**
(R^2)	-0.1000	0.9000	1.7200	1.7100	0.9800
SMB	0.0011	-0.0004	-0.0005	-0.0009	-0.0009
(R^2)	0.0100	-0.1100	-0.0800	0.0400	0.1000
HML	-0.0003	-0.0003	0.0001	-0.0001	-0.0004
(R^2)	-0.1100	-0.1100	-0.1200	-0.1200	-0.1000
MOM	-0.00360**	-0.0018	-0.0008	0.0000	0.0001
(R^2)	2.4600	0.6500	0.0500	-0.1200	-0.1200

Predictability of Market Volatilities

	T=0	T=0.5	T=1	T=2	T=3
Mkt	0.0003	-0.0019	-0.0024**	-0.0021***	-0.0014**
(R^2)	-0.1000	0.9000	1.7200	1.7100	0.9800
MPC1	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
(R^2)	-0.0300	-0.0400	-0.0600	0.0200	0.1100
MRPC1	-0.00225***	-0.0010	-0.0003	0.0001	0.0002
(R^2)	1.6800	0.3000	-0.0800	-0.1200	-0.1000
SPC1	0.0001	-0.0001	-0.0002	-0.0002	-0.0002
(R^2)	-0.1000	-0.1100	0.1000	0.1800	0.2300
SRPC1	0.0001	0.0000	0.0001	0.0001	0.0000
(R^2)	-0.1200	-0.1200	-0.1200	-0.1200	-0.1300

Unconditional Factor Model Test

- Time-series regression:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{n} \beta_{j,i} F_{j,t} + \epsilon_{i,t}$$

  where $R_{i,t}$ is the excess return of test assets

- Cross-sectional regression:

$$E(R_i) = (\gamma) + \sum_{j=1}^{n} \beta_{j,i} \lambda_j + \alpha_i$$

Time-series Regression: Momentum

Alpha	F-test	N(***)	N(**)	N(*)
CAPM	6.89***	5	5	7
FF4	4.04***	2	4	4
DCB(1 RPC)	3.35***	2	4	5
DCB(3 RPCs)	2.46***	1	3	5
DCB(1 PC)	7.03***	5	5	7
DCB(3 PCs)	7.10***	5	5	7
FF4 & RPC	3.92***	2	5	5
FF4 & PC	4.14***	2	4	4

Cross-sectional Regression without constant: Momentum

λ	CAPM	FF4	1RPC	1PC	3RPCs	3PCs
Mkt	6.56***	14.16***	8.15***	8.84***	8.34***	8.27***
SMB	-	-30.26*	-	-	-	-
HML	-	-53.96*	-	-	-	-
MOM	-	7.40***	-	-	-	-
RPC1	-	-	13.07***	-	13.50***	-
RPC2	-	-	-	-	-3.78***	-
RPC3	-	-	-	-	-2.43***	-
PC1	-	-	-	222.9***	-	140.93*
PC2	-	-	-	-	-	-120.03*
PC3	-	-	-	-	-	-11.42
Chi^2	61.56***	3.59	25.56***	15.28**	15.51***	7.49
R^2	45.51	94.17	83.89	83.89	95.61	95.61

Cross-sectional Regression without constant: Momentum

λ	FRPC	FPC
Mkt	12.72**	13.26***
SMB	-23.90	-23.87
HML	-41.31	-41.88
MOM	7.04**	8.24**
RPC1	13.55***	-
RPC2	-	-
RPC3	-	-
PC1	-	184.6
PC2	-	-
PC3	-	-
Chi^2	5.25	4.63
R^2	94.43	94.43

Cross-sectional Regression without constant: Momentum

Alpha	N(***)	N(**)	N(*)
CAPM	5	6	7
FF4	0	0	0
DCB(1 RPC)	2	4	5
DCB(3 RPCs)	0	3	6
DCB(1 PC)	0	2	4
DCB(3 PCs)	0	0	2
FF4 & RPC	0	0	1
FF4 & PC	0	0	0

Cross-sectional Regression with constant: Momentum

λ	CAPM	FF4	1RPC	1PC	3RPCs	3PCs
Mkt	-12.82**	-11.41	-0.79	-0.19	-7.00	-4.99
SMB	-	-13.17	-	-	-	-
HML	-	-43.09*	-	-	-	-
MOM	-	7.99***	-	-	-	-
RPC1	-	-	11.50***	-	11.12***	-
RPC2	-	-	-	-	-2.87**	-
RPC3	-	-	-	-	-3.94***	-
PC1	-	-	-	193.3***	-	144.47*
PC2	-	-	-	-	-	57.96
PC3	-	-	-	-	-	-61.63*
γ	20.60***	24.57*	9.28**	9.28*	15.88**	15.88**
Chi^2	43.70***	2.56	16.58**	10.89	4.28	2.09
R^2	13.18	97.88	93.65	93.65	98.86	98.86

Cross-sectional Regression with constant: Momentum

λ	FRPC	FPC
Mkt	-14.92	-14.1
SMB	-2.64	-2.6
HML	-23.38	-24.24
MOM	7.49***	9.30***
RPC1	10.88***	-
RPC2	-	-
RPC3	-	-
PC1	-	259.79
PC2	-	-
PC3	-	-
γ	25.85**	25.85*
Chi^2-test	3.91	2.48
R^2	98.47	98.47

Cross-sectional Regression with constant: Momentum

Alpha	N(***)	N(**)	N(*)
CAPM	2	4	9
FF4	0	0	0
DCB(1 RPC)	1	3	5
DCB(3 RPCs)	0	0	0
DCB(1 PC)	0	2	4
DCB(3 PCs)	0	0	0
FF4 & RPC	0	0	0
FF4 & PC	0	0	0

Time-series Regression: SizeBook

Alpha	F-test	N(***)	N(**)	N(*)
CAPM	4.15***	9	13	15
FF4	2.66***	4	8	8
DCB(2 RPCs)	3.78***	6	9	10
DCB(2 PCs)	4.19***	9	13	15
FF4 & RPCs	2.55***	4	8	8
FF4 & PCs	2.71***	5	8	8

Cross-sectional Regression without constant: SizeBook

λ	CAPM	FF4	2RPCs	2PCs	FRPC	FPC
Mkt	8.61***	8.21***	7.84***	7.68***	7.88***	7.72***
SMB	-	1.22	-	-	2.06	2.02
HML	-	4.78***	-	-	4.37***	4.31***
MOM	-	36.97***	-	-	49.19***	48.70***
RPC1	-	-	1.6	-	-4.85	-
RPC2	-	-	9.43***	-	5.81***	-
PC1	-	-	-	149.2*	-	-21.68
PC2	-	-	-	134.3***	-	109.5**
Chi^2	102.6***	33.46**	88.07***	39.63***	20.9	16.31
R^2	70.25	67.21	28.78	28.78	81.54	81.54

Cross-sectional Regression without constant: SizeBook

Alpha	N(***)	N(**)	N(*)
CAPM	10	13	13
FF4	1	2	7
DCB(2 RPCs)	7	11	13
DCB(2 PCs)	1	7	9
FF4 & RPCs	0	1	4
FF4 & PCs	0	0	2

Cross-sectional Regression with constant: SizeBook

λ	CAPM	FF4	2RPCs	2PCs	FRPC	FPC
Mkt	-6.88	-1.31	-7.32*	-7.41	5.09	4.93
SMB	-	1.13	-	-	1.98	1.95
HML	-	4.49***	-	-	4.30***	4.25***
MOM	-	24.01***	-	-	44.71***	44.23***
RPC1	-	-	9.87**	-	-3.64	-17.86
RPC2	-	-	3.55	-	5.26*	103.43**
PC1	-	-	-	163.65**	-	-
PC2	-	-	-	86.69**	-	-
γ	16.77***	9.47**	15.87***	15.87***	2.8	2.8
Chi^2	66.83***	36.05**	60.87***	33.71**	20.60	16.06
R^2	11.84	70.84	47.86	47.86	81.81	81.81

Cross-sectional Regression with constant: SizeBook

Alpha	N(***)	N(**)	N(*)
CAPM	7	13	14
FF4	3	5	9
DCB(2 RPCs)	5	10	12
DCB(2 PCs)	2	5	9
FF4 & RPCs	0	2	4
FF4 & PCs	0	0	3

Conditional Factor Model Test

- Orthogonalize factors using GARCH:

$$F_{2,t} = c_0 + c_{1,t} F_{1,t} + F^*_{2,t}$$

then we use the set of orthogonalized factors F^* to construct conditional betas

- Estimate conditional beta through GARCH model:

$$\beta_{j,i,t} = \frac{\sigma_{ij,t}}{\sigma^2_{j,t}} = \rho_{ij,t} \frac{\sigma_{i,t}}{\sigma_{j,t}}$$

- Fama-MacBeth Procedure using conditional betas:

$$R_{i,t} = (\gamma_t) + \sum_{j=1}^{n} \beta_{j,i,t} \lambda_{j,t} + \alpha_{i,t}$$

$$\lambda_j = \frac{1}{T} \sum_{t=1}^{T} \lambda_{j,t}, \quad \alpha_i = \frac{1}{T} \sum_{t=1}^{T} \alpha_{i,t}$$

Conditional without constant: Momentum

λ	CAPM	FF4	1RPC	1PC	FRPC	FPC
Mkt	6.69***	9.37***	7.76***	8.80***	8.40***	9.65***
SMB	-	-11.53**	-	-	-12.80**	-12.67**
HML	-	-14.55**	-	-	-11.38*	-14.25**
MOM	-	4.28	-	-	3.7	4.49
RPC1	-	-	15.09***	-	5.39**	34.59
PC1	-	-	-	335.4***	-	-
Chi^2	68.30***	35.37***	28.47***	36.21***	34.99***	42.90***
R^2	2.08	64.7	41.14	39.65	70.95	70.4

Conditional without constant: Momentum

Alpha	N(***)	N(**)	N(*)
CAPM	5	6	6
FF4	2	3	5
DCB(1 RPC)	1	4	4
DCB(1 PC)	1	5	5
FF4 & RPC	0	3	4
FF4 & PC	2	3	4

Conditional with constant: Momentum

λ	CAPM	FF4	1RPC	1PC	FRPC	FPC
Mkt	0.69	-4.89	-3.58	-4.01	-9.26*	-5.85
SMB	-	-6.12	-	-	-7.96	-8.89
HML	-	-10.10*	-	-	-7.44	-13.82*
MOM	-	8.48**	-	-	6.27*	7.16*
RPC1	-	-	15.80***	-	6.00**	41.09
PC1	-	-	-	351.17***	-	-
γ	20.60***	24.57*	9.28**	25.85**	9.28*	25.85*
Chi^2	68.03***	29.90***	14.79*	22.84***	16.66*	35.22***
R^2	28.82	72.57	56.32	55.96	78.56	77.96

Conditional with constant: Momentum

Alpha	N(***)	N(**)	N(*)
CAPM	3	5	5
FF4	2	3	3
DCB(1 RPC)	0	1	3
DCB(1 PC)	0	3	5
FF4 & RPC	0	3	3
FF4 & PC	2	4	4

Conditional without constant: SizeBook

λ	CAPM	FF4	RPC	PC	FRPC	FPC
Mkt	8.39***	9.08***	8.51***	8.84***	8.77***	9.39***
SMB	-	2.65*	-	-	3.13**	2.76*
HML	-	4.00***	-	-	4.10***	3.96***
MOM	-	18.67***	-	-	19.83***	19.97***
RPC1	-	-	1.23	-	-1.86	-
RPC2	-	-	7.24***	-	3.25**	-
PC1	-	-	-	-17.23	-	18.08
PC2	-	-	-	44.44**	-	10.35
Chi^2	109.8***	91.86***	103.1***	103.5***	93.94***	89.88***
R^2	6.33	49.39	36.63	31.52	56.88	56.29

Conditional without constant: SizeBook

Alpha	N(***)	N(**)	N(*)
CAPM	10	12	13
FF4	3	5	6
DCB(2 RPCs)	7	13	14
DCB(2 PCs)	5	6	7
FF4 & RPC	6	10	13
FF4 & PC	3	5	6

Conditional with constant: SizeBook

λ	CAPM	FF4	RPC	PC	FRPC	FPC
Mkt	-4.52	-3.89	-4.12	-3.43	-3.26	-3.24
SMB	-	3.10**	-	-	3.51**	3.12**
HML	-	2.85**	-	-	2.92**	2.89**
MOM	-	17.91***	-	-	18.39***	18.41***
RPC1	-	-	4.64	-	-1.55	-
RPC2	-	-	2.93*	-	2.1	-
PC1	-	-	-	-3.11	-	13.6
PC2	-	-	-	15.48	-	6.75
γ	16.77***	9.47**	15.87***	15.87***	2.8	2.8
Chi^2	65.73***	51.09***	72.17***	66.50***	53.34***	48.63***
R^2	21.97	54.02	42.82	39.45	60.98	60.37

Conditional with constant: SizeBook

Alpha	N(***)	N(**)	N(*)
CAPM	6	10	12
FF4	2	4	5
DCB(2 RPCs)	4	9	10
DCB(2 PCs)	5	7	9
FF4 & RPC	3	4	6
FF4 & PC	3	3	5

Conclusion

- We use a multivariate GARCH approach to construct the conditional market beta for different assets
- We use Principle Component Analysis (PCA) to extract the covariation of conditional betas and constructed factor mimicking portfolios
- There is some predictability of PC factors in market returns and market volatilities$\rightarrow$ hints to be a state variable
- The correlation between PC factors and Fama-French is large
- PC factors earns large abnormal returns according to popular factor pricing models $\rightarrow$ pricing factors miss important risks
- Alphas in CAPM is larger than alphas FF4$\rightarrow$ some overlap between Fama-French factors and PCs
- As a pricing factor PC factors can explain some of the data compared to other popular asset pricing models

Future Steps

- Look into the conditional test results
- Run the tests across different asset sets
- Conditional models are often rejected
    - Other factors?
    - Estimation errors?