Bank Runs, Deposit Insurance, and Liquidity
More Technical Details

Douglas W. Diamond
University of Chicago
Philip H. Dybvig
Washington University in Saint Louis

Southwest University of Finance and Economics

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Free download of a clean reprint at *http://minneapolisfed.org/research/qr/qr2412.pdf*.

Modern Problem Formulation

I would ask a PhD student with this sort of model to write it up a little more formally than we did in the paper. For example, there should be a choice variable for whether to withdraw. This sounds complicated, but strangely it usually makes the paper shorter and easier to understand.

Competitive Model: Choice Problem

Choose nonnegative c_1^1, c_1^2, c_2^1, c_2^2, and I, and choose L_1 and L_2 in $[0,1]$, to maximize $tu(c_1^1) + (1-t)\rho u(c_1^2 + c_2^2)$
subject to:

$$p_1 c_1^1 + p_2 c_2^1 = p_0(1-I) + p_1 I L_1 + p_2 I(1-L_1)R$$

and

$$p_1 c_1^2 + p_2 c_2^2 = p_0(1-I) + p_1 I L_2 + p_2 I(1-L_2)R$$

Market clearing: $\int I = 1$, $\int (tL_1 + (1-t)L_2)I = \int (tc_1^1 + (1-t)c_1^2)$, and
$R \int (1 - (tL_1 + (1-t)L_2))I = \int (tc_2^1 + (1-t)c_2^2)$

Equilibrium: $p_0 = 1$, $p_1 = 1$, $p_2 = R^{-1}$
$c_1^1 = 1$, $c_2^2 = R$, $c_1^2 = c_2^1 = 0$
I many solutions, all have total investment $\int I = 1$
L_1,L_2 many solutions, all have total liquidation $\int (tL_1 + (1-t)L_2)I = t$

same consumption as autarky solution $I = 1$, $L_1 = 1$, $L_2 = 0$

Full Information Optimal Solution

Assume (correctly) a symmetric solution.

Choose nonnegative c_1^1, c_1^2, c_2^1, and c_2^2 to
maximize $tu(c_1^1) + (1-t)\rho u(c_1^2 + c_2^2)$
subject to:
$(tc_1^1 + (1-t)c_1^2) + (tc_2^1 + (1-t)c_2^2)/R = 1$

Solution:
$c_1^2 = c_2^1 = 0$

first-order condition:
$u'(c_1^{1*}) = \rho R u'(c_2^{2*})$

Since $\rho R > 1$, $c_2^{2*} > c_1^{1*}$. Also, $RRA > 1$ implies $c_1^{1*} > 1$ and $c_2^{2*} < R$

Bank Deposit Contract Payoff

In the bank deposit contract, the bank promises r_1 to each depositor who withdraws in period 1. Depositors arrive sequentially, each with a uniform distribution over place in line, and the bank liquidates assets as necessary to pay each depositor until assets are exhausted. Once assets are exhausted, the bank fails and all remaining depositors receive nothing (whether or not they withdrew).

If bank assets are not exhausted, then all depositors who do not withdraw share equally in the assets in the last period. This "mutual bank" assumption avoids the necessity of modelling another agent (the bank owner) and avoids any issues of industrial organization that are not part of what we want to study.

Note: the bank deposit contract satisfies the sequential service constraint.

Bank Depositor Withdrawal Choice Problem

For type 1 agents, withdrawing is a dominant strategy because not withdrawing implies consumption is always 0 but withdrawing implies a positive probability of consuming a positive amount. So we can reasonably assume that type 1 agents always withdraw.[1] Note that the fraction of all agents who withdraw is therefore $f = t + (1-t)f_2 \in [t, 1]$ where $f_2 \in [0, 1]$ is the fraction of type 2 agents who withdraw.

Then, letting W_2 be the withdrawal choice of a type 2 depositor, the type 2 depositor's objective function for $f < 1$ is

$$\max(1-1/(fr_1), 0)\rho u(0)+\min(1/(fr_1), 1)\rho u(W_2 r_1+(1-W_2)\max(\tfrac{1-fr_1}{1-f}R, 0)).$$

[1] There are examples when elimiation of dominated strategies can lead to strange results, but not so in this model.

Payoffs When $f = 1$

For $f = 1$, the factor $1 - f$ in the denominator indicates there is a problem and in fact fixing the problem is a little subtle. If $r_1 < 1$, the payoff from waiting is infinite because a single infinitessimal agent gets claim to a non-infinitessimal residual in the bank. For $r_1 = 1$, waiting pays off R because not withdrawing leaves just the agent's own claim in the bank, and the agent will optimally choose not to withdraw. This observation can be used to show that $r_1 = 1$ leads to the autarky solution. The remaining case, when $r_1 > 1$, is the normal case and the most interesting. In this case, $f = 1$ exhausts the bank's assets even if the agent under consideration does not withdraw, and the payoff for $f = 1$ and $r_1 > 1$ is therefore

$$\max(1 - 1/(fr_1), 0)\rho u(0) + \min(1/(fr_1), 1)\rho u(W_2 r_1).$$

Banking Equilibria

For $1 < r_1 \leq c_1^{1*}$, there are two types of pure strategy equilibria:

no run: $W_1 = 1$, $W_2 = 0$, and $f = t$
run: $f = W_1 = W_2 = 1$

When $r_1 = c_1^{1*}$, then $\frac{1-fr_1}{1-f}R = c_2^{2*}$ and the no-run equilibrium is first-best.

technical note: we want to assume $u(0)$ finite and $(\forall c \in [1, R]) - cu''(c)/u'(c) > 1$. (different than the paper)

A Technical Comment

Measure theoretic issue. emphasized by Ken Judd, nice solution by Bob Anderson

Deposit Insurance

Let's focus on $r_1 = c_1^{1^*}$. It would be nice if we could design a policy that eliminates the run equilibrium without disrupting the optimal no-run equilibrium. In fact, a guarantee to people that wait that they will get money back when they wait (perhaps backed by seignorage and/or taxation authority) will do so. Let the guarantee be $G \in (r_1, R]$. A guarantee in period 2 does not affect the incentives of a type 1 agent. For a type 2 agent, the payoff if $f < 1$ becomes

$$\max(1-1/(fr_1), 0)\rho u(0) + \min(1/(fr_1), 1)\rho u(W_2 r_1 + (1-W_2)\max(\tfrac{1-fr_1}{1-f}R, G)),$$

which is decreasing in W_2. Similarly, if $f = 1$ the payoff is

$$\max(1 - 1/(fr_1), 0)\rho u(0) + \min(1/(fr_1), 1)\rho u(W_2 r_1 + (1 - W_2)G),$$

which is also decreasing in W_2.

Discount Window, Practical Considerations

Use of riskless borrowing at the discount window could also prevent runs, but the policy for using the discount window would have to be designed carefully. If unlimited borrowing is available at a low rate, there is an arbitrage and the discount window could be used to finance investment. However, if the rate is high it will not help the bank any (assuming the bank will repay the borrowing). So, the central bank will probably need to use discretion in deciding there is a run before lending, but in this case maybe it is not credible that the discount window will necessarily be available when the bank needs it. For example, the central bank might decide the bank is unsound and refuse access to the discount window.

Note that deposit insurance costs the guarantor nothing in our model, but in practice risky assets would make deposit insurance costly so that the guarantor would have to have some type of incentive scheme and monitoring to guard against risky assets.

Suspension of Convertibility, Sequential Service, Random t

Suspension of convertibility can stop a run (for example, if only the first t depositors are payed off in period 1 and the rest have to wait), but such a rigid policy can do a lot of damage if t is random.

With sequential service and random t, in general it is optimal to offer a contract that pays more to early withdrawers. If it were possible, it would be nice to wait and see how many total withdrawers arrive before deciding how much money to give everyone, but we think that is impractical.